CHAPTER 43
HEAT TRANSFER FUNDAMENTALS

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43.1 SYMBOLS AND UNITS

A	area of heat transfer
Bi	Biot number, hL/k, dimensionless
C	circumference, m, constant defined in text
C_p	specific heat under constant pressure, J/kg · K
D	diameter, m
e	emissive power, W/m^2
f	drag coefficient, dimensionless
F	cross flow correction factor, dimensionless
F_{i-j}	configuration factor from surface i to surface j, dimensionless
Fo	Fourier number, $\alpha t A^2/V^2$, dimensionless
$F_{o-\lambda T}$	radiation function, dimensionless
G	irradiation, W/m^2; mass velocity, kg/m^2 · sec
g	local gravitational acceleration, 9.8 m/sec^2
g_c	proportionality constant, 1 kg · m/N · sec^2
Gr	Grashof number, $gL^3\beta\Delta t/v^2$, dimensionless
h	convection heat transfer coefficient, equals $q/A\Delta T$, W/m^2 · K
h_{fg}	heat of vaporization, J/kg
J	radiocity, W/m^2
k	thermal conductivity, W/m · K

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K	wick permeability, m^2
L	length, m
Ma	Mach number, dimensionless
N	screen mesh number, m^{-1}
Nu	Nusselt number, $Nu_L = hL/k$, $Nu_D = hD/k$, dimensionless
$\overline{\text{Nu}}$	Nusselt number averaged over length, dimensionless
P	pressure, N/m^2, perimeter, m
Pe	Peclet number, RePr, dimensionless
Pr	Prandtl number, $C_p\mu/k$, dimensionless
q	rate of heat transfer, W
q''	rate of heat transfer per unit area, W/m^2
R	distance, m; thermal resistance, K/W
r	radial coordinate, m; recovery factor, dimensionless
Ra	Rayleigh number, GrPr; $Ra_L = Gr_L Pr$, dimensionless
Re	Reynolds number, $Re_L = \rho VL/\mu$, $Re_D = \rho VD/\mu$, dimensionless
S	conduction shape factor, m
T	temperature, K or °C
t	time, sec
T_{as}	adiabatic surface temperature, K
T_{sat}	saturation temperature, K
T_b	fluid bulk temperature or base temperature of fins, K
T_e	excessive temperature, $T_s - T_{sat}$, K or °C
T_f	film temperature, $(T_\infty + T_s)/2$, K
T_i	initial temperature; at $t = 0$, K
T_o	stagnation temperature, K
T_s	surface temperature, K
T_∞	free stream fluid temperature, K
U	overall heat transfer coefficient, $W/m^2 \cdot K$
V	fluid velocity, m/sec; volume, m^3
w	groove width, m; or wire spacing, m
We	Weber number, dimensionless
x	one of the axes of Cartesian reference frame, m

Greek Symbols

α	thermal diffusivity, $k/\rho C_p$, m^2/sec; absorptivity, dimensionless
β	coefficient of volume expansion, 1/K
Γ	mass flow rate of condensate per unit width, $kg/m \cdot sec$
γ	specific heat ratio, dimensionless
ΔT	temperature difference, K
δ	thickness of cavity space, groove depth, m
ϵ	emissivity, dimensionless
ε	wick porosity, dimensionless
λ	wavelength, μm
η_f	fin efficiency, dimensionless
μ	viscosity, $kg/m \cdot sec$
ν	kinematic viscosity, m^2/sec
ρ	reflectivity, dimensionless; density, kg/m^3
σ	surface tension, N/m; Stefan–Boltzmann constant, $5.729 \times 10^{-8}\ W/m^2 \cdot K^4$
τ	transmissivity, dimensionless, shear stress, N/m^2
Ψ	angle of inclination, degrees or radians

Subscripts

a	adiabatic section, air
b	boiling, black body
c	convection, capillary, capillary limitation, condenser
e	entrainment, evaporator section
eff	effective
f	fin
i	inner
l	liquid
m	mean, maximum
n	nucleation
o	outer
0	stagnation condition
p	pipe
r	radiation

s	surface, sonic or sphere
w	wire spacing, wick
v	vapor
λ	spectral
∞	free stream
$-$	axial hydrostatic pressure
$+$	normal hydrostatic pressure

The science or study of heat transfer is that subset of the larger field of transport phenomena that focuses on the energy transfer occurring as a result of a temperature gradient. This energy transfer can manifest itself in several forms, including *conduction,* which focuses on the transfer of energy through the direct impact of molecules; *convection,* which results from the energy transferred through the motion of a fluid; and *radiation,* which focuses on the transmission of energy through electromagnetic waves. In the following review, as is the case with most texts on heat transfer, *phase change heat transfer,* that is, *boiling* and *condensation,* will be treated as a subset of convection heat transfer.

43.2 CONDUCTION HEAT TRANSFER

The exchange of energy or heat resulting from the kinetic energy transferred through the direct impact of molecules is referred to as *conduction* and takes place from a region of high energy (or temperature) to a region of lower energy (or temperature). The fundamental relationship that governs this form of heat transfer is *Fourier's law of heat conduction,* which states that in a one-dimensional system with no fluid motion, the rate of heat flow in a given direction is proportional to the product of the temperature gradient in that direction and the area normal to the direction of heat flow. For conduction heat transfer in the x-direction this expression takes the form

$$q_x = -kA \frac{\partial T}{\partial x}$$

where q_x is the heat transfer in the x-direction, A is the area normal to the heat flow, $\partial T/\partial x$ is the temperature gradient, and k is the thermal conductivity of the substance.

Writing an energy balance for a three-dimensional body, and utilizing Fourier's law of heat conduction, yields an expression for the transient diffusion occurring within a body or substance.

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial}{\partial x} \frac{\partial T}{\partial t}$$

This expression, usually referred to as the *heat diffusion equation* or heat equation, provides a basis for most types of heat conduction analysis. Specialized cases of this equation, such as the case where the thermal conductivity is a constant

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho c_p}{k} \frac{\partial T}{\partial t}$$

steady-state with heat generation

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \dot{q} = 0$$

steady-state, one-dimensional heat transfer with heat transfer to a heat sink (i.e., a fin)

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\dot{q}}{k} = 0$$

or one-dimensional heat transfer with no internal heat generation

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\rho c_p}{k} \frac{\partial T}{\partial t}$$

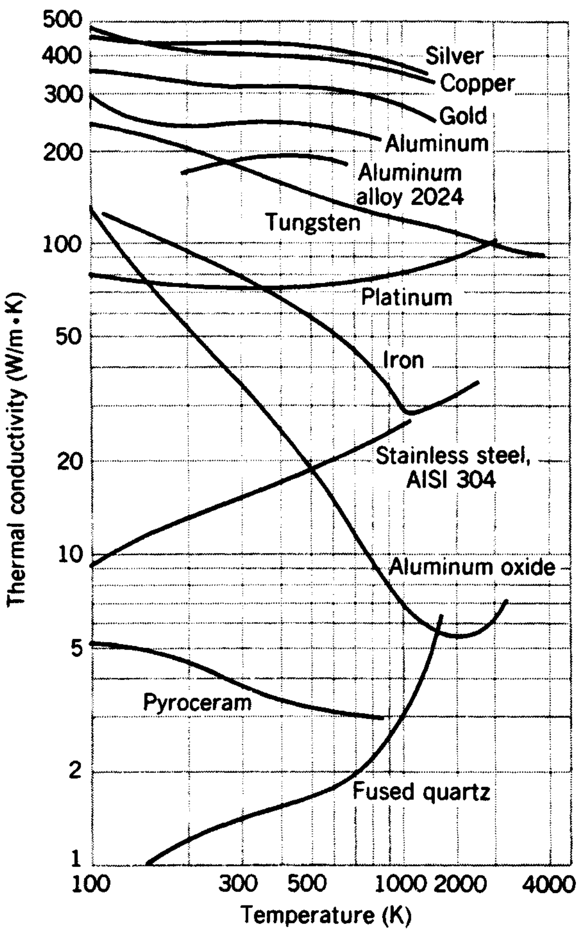
can be utilized to solve many steady-state or transient problems. In the following sections, this equation will be utilized for several specific cases. However, in general, for a three-dimensional body of constant thermal properties without heat generation under steady-state heat conduction, the temperature field satisfies the expression

$$\nabla^2 T = 0$$

43.2.1 Thermal Conductivity

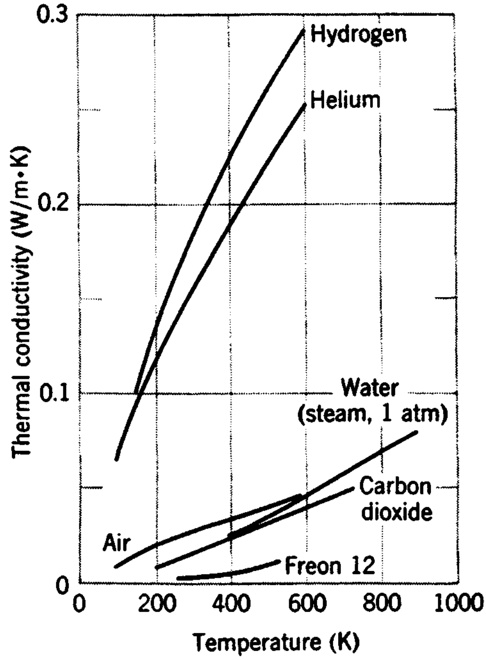
The ability of a substance to transfer heat through conduction can be represented by the constant of proportionality *k*, referred to as the thermal conductivity. Figures 43.1*a*, *b*, and *c* illustrate the characteristics of the thermal conductivity as a function of temperature for several solids, liquids and gases, respectively. As shown, the thermal conductivity of solids is higher than liquids, and liquids higher than gases. Metals typically have higher thermal conductivities than nonmetals, with pure metals having thermal conductivities that decrease with increasing temperature, while the thermal conductivities of nonmetallic solids generally increase with increasing temperature and density. The addition of other metals to create alloys, or the presence of impurities, usually decreases the thermal conductivity of a pure metal.

In general, the thermal conductivities of liquids decrease with increasing temperature. Alternatively, the thermal conductivities of gases and vapors, while lower, increase with increasing temperature and decrease with increasing molecular weight. The thermal conductivities of a number of commonly used metals and nonmetals are tabulated in Tables 43.1 and 43.2, respectively. Insulating materials, which are used to prevent or reduce the transfer of heat between two substances or a substance and the surroundings are listed in Tables 43.3 and 43.4, along with the thermal properties. The thermal conductivities for liquids, molten metals, and gases are given in Tables 43.5, 43.6 and 43.7, respectively.



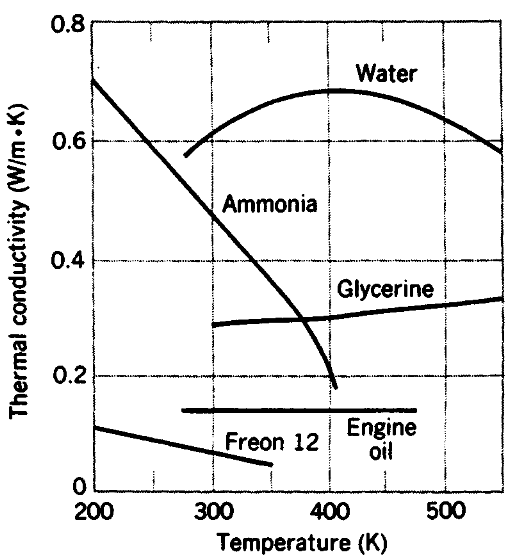
(*a*)

Fig. 43.1a Temperature dependence of the thermal conductivity of selected solids.



(*b*)

Fig. 43.1b Selected nonmetallic liquids under saturated conditions.



(*c*)

Fig. 43.1c Selected gases at normal pressures.[1]

Table 43.1 Thermal Properties of Metallic Solids[a]

Composition	Melting Point (K)	Properties at 300 K				Properties at Various Temperatures (K) k(W/m · K); C_p(J/kg · K)		
		ρ (kg/m^3)	C_p (J/kg · K)	k (W/m · K)	$\alpha \times 10^6$ (m^2/sec)	100	600	1200
Aluminum	933	2702	903	237	97.1	302; 482	231; 1033	
Copper	1358	8933	385	401	117	482; 252	379; 417	339; 480
Gold	1336	19300	129	317	127	327; 109	298; 135	255; 155
Iron	1810	7870	447	80.2	23.1	134; 216	54.7; 574	28.3; 609
Lead	601	11340	129	35.3	24.1	39.7; 118	31.4; 142	
Magnesium	923	1740	1024	156	87.6	169; 649	149; 1170	
Molybdenum	2894	10240	251	138	53.7	179; 141	126; 275	105; 308
Nickel	1728	8900	444	90.7	23.0	164; 232	65.6; 592	76.2; 594
Platinum	2045	21450	133	71.6	25.1	77.5; 100	73.2; 141	82.6; 157
Silicon	1685	2330	712	148	89.2	884; 259	61.9; 867	25.7; 967
Silver	1235	10500	235	429	174	444; 187	412; 250	361; 292
Tin	505	7310	227	66.6	40.1	85.2; 188		
Titanium	1953	4500	522	21.9	9.32	30.5; 300	19.4; 591	22.0; 620
Tungsten	3660	19300	132	174	68.3	208; 87	137; 142	113; 152
Zinc	693	7140	389	116	41.8	117; 297	103; 436	

[a]Adapted from F. P. Incropera and D. P. Dewitt, *Fundamentals of Heat Transfer*. © 1981 John Wiley & Sons, Inc. Reprinted by permission.

Table 43.2 Thermal Properties of Nonmetals

Description/Composition	Temperature (K)	Density ρ (kg/m^3)	Thermal Conductivity k ($W/m \cdot K$)	Specific Heat C_p ($J/kg \cdot K$)	$\alpha \times 10^6$ (m^2/sec)
Bakelite	300	1300	0.232	1465	0.122
Brick, refractory					
Carborundum	872	—	18.5	—	—
Chrome-brick	473	3010	2.32	835	0.915
Fire clay brick	478	2645	1.0	960	0.394
Clay	300	1460	1.3	880	1.01
Coal, anthracite	300	1350	0.26	1260	0.153
Concrete (stone mix)	300	2300	1.4	880	0.692
Cotton	300	80	0.059	1300	0.567
Glass, window	300	2700	0.78	840	0.344
Rock, limestone	300	2320	2.15	810	1.14
Rubber, hard	300	1190	0.160	—	—
Soil, dry	300	2050	0.52	1840	0.138
Teflon	300	2200	0.35	—	—
	400	—	0.45	—	—

Table 43.3 Thermal Properties of Building and Insulating Materials (at 300K)[a]

Description/Composition	Density ρ (kg/m^3)	Thermal Conductivity k ($W/m \cdot K$)	Specific Heat C_p ($J/kg \cdot K$)	$\alpha \times 10^6$ (m^2/sec)
Building boards				
Plywood	545	0.12	1215	0.181
Acoustic tile	290	0.058	1340	0.149
Hardboard, siding	640	0.094	1170	0.126
Woods				
Hardwoods (oak, maple)	720	0.16	1255	0.177
Softwoods (fir, pine)	510	0.12	1380	0.171
Masonry materials				
Cement mortar	1860	0.72	780	0.496
Brick, common	1920	0.72	835	0.449
Plastering materials				
Cement plaster, sand aggregate	1860	0.72	—	—
Gypsum plaster, sand aggregate	1680	0.22	1085	0.121
Blanket and batt				
Glass fiber, paper faced	16	0.046	—	—
Glass fiber, coated; duct liner	32	0.038	835	1.422
Board and slab				
Cellular glass	145	0.058	1000	0.400
Wood, shredded/cemented	350	0.087	1590	0.156
Cork	120	0.039	1800	0.181
Loose fill				
Glass fiber, poured or blown	16	0.043	835	3.219
Vermiculite, flakes	80	0.068	835	1.018

[a]Adapted from F. P. Incropera and D. P. Dewitt, *Fundamentals of Heat Transfer*. © 1981 John Wiley & Sons, Inc. Reprinted by permission.

Table 43.4 Thermal Conductivities for Some Industrial Insulating Materials[a]

Description/Composition	Maximum Service Temperature (K)	Typical Density (kg/m^3)	Typical Thermal Conductivity, k (W/m · K), at Various Temperatures (K)			
			200	300	420	645
Blankets						
Blanket, mineral fiber, glass; fine fiber organic bonded	450	10		0.048		
		48		0.033		
Blanket, alumina–silica fiber	1530	48				0.105
Felt, semirigid; organic bonded	480	50–125		0.038	0.063	
Felt, laminated; no binder	920	120			0.051	0.087
Blocks, boards, and pipe insulations						
Asbestos paper, laminated and corrugated, 4-ply	420	190		0.078		
Calcium silicate	920	190			0.063	0.089
Polystyrene, rigid						
Extruded (R-12)	350	56	0.023	0.027		
Molded beads	350	16	0.026	0.040		
Rubber, rigid foamed	340	70		0.032		
Insulating cement						
Mineral fiber (rock, slag, or glass)						
With clay binder	1255	430			0.088	0.123
With hydraulic setting binder	922	560			0.123	
Loose fill						
Cellulose, wood or paper pulp	—	45		0.039		
Perlite, expanded	—	105	0.036	0.053		
Vermiculite, expanded	—	122		0.068		

[a]Adapted from F. P. Incropera and D. P. Dewitt, *Fundamentals of Heat Transfer*. © 1981 John Wiley & Sons, Inc. Reprinted by permission.

Table 43.5 Thermal Properties of Saturated Liquids[a]

T (K)	ρ (kg/m^3)	C_p ($kJ/kg \cdot K$)	$\nu \times 10^6$ (m^2/sec)	$k \times 10^3$ ($W/m \cdot K$)	$\alpha \times 10^7$ (m^2/sec)	Pr	$\beta \times 10^3$ (K^{-1})
Ammonia, Nh_3							
223	703.7	4.463	0.435	547	1.742	2.60	2.45
323	564.3	5.116	0.330	476	1.654	1.99	2.45
Carbon Dioxide, CO_2							
223	1,156.3	1.84	0.119	85.5	0.402	2.96	14.0
303	597.8	36.4	0.080	70.3	0.028	28.7	14.0
Engine Oil (Unused)							
273	899.1	1.796	4,280	147	0.910	47,000	0.70
430	806.5	2.471	5.83	132	0.662	88	0.70
Ethylene Glycol, $C_2H_4(OH)_2$							
273	1,130.8	2.294	57.6	242	0.933	617.0	0.65
373	1,058.5	2.742	2.03	263	0.906	22.4	0.65
Clycerin, $C_3H_5(OH)_3$							
273	1,276.0	2.261	8,310	282	0.977	85,000	0.47
320	1,247.2	2.564	168	287	0.897	1,870	0.50
Freon (Refrigerant-12), CCl_2F_2							
230	1,528.4	0.8816	0.299	68	0.505	5.9	1.85
320	1,228.6	1.0155	0.190	68	0.545	3.5	3.50

[a]Adapted from Ref. 2. See Table 43.23 for H_2O.

43.2.2 One-Dimensional Steady-State Heat Conduction

The rate of heat transfer for steady-state heat conduction through a homogeneous material can be expressed as $q = \Delta T/R$, where ΔT is the temperature difference and R is the *thermal resistance.* This thermal resistance, is the reciprocal of the *thermal conductance* ($C = 1/R$) and is related to the thermal conductivity by the cross-sectional area. Expressions for the thermal resistance, the temperature distribution, and the rate of heat transfer are given in Table 43.8 for a plane wall, a cylinder, and a sphere. For the plane wall, the heat transfer is assumed to be one-dimensional (i.e., conducted only in the x-direction) and for the cylinder and sphere, only in the radial direction.

In addition to the heat transfer in these simple geometric configurations, another common problem encountered in practice is the heat transfer through a layered or composite wall consisting of N layers where the thickness of each layer is represented by Δx_n and the thermal conductivity by k_n for $n = 1, 2, \ldots, N$. Assuming that the interfacial resistance is negligible (i.e., there is no thermal resistance at the contacting surfaces), the overall thermal resistance can be expressed as

$$R = \sum_{n=1}^{N} \frac{\Delta x_n}{k_n A}$$

Similarly, for conduction heat transfer in the radial direction through N *concentric cylinders* with negligible interfacial resistance, the overall thermal resistance can be expressed as

$$R = \sum_{n=1}^{N} \frac{\ln(r_{n+1}/r_n)}{2\pi k_n L}$$

where r_1 = inner radius
r_{N+1} = outer radius

For N *concentric spheres* with negligible interfacial resistance, the thermal resistance can be expressed as

$$R = \sum_{n=1}^{N} \left(\frac{1}{r_n} - \frac{1}{r_{n+1}}\right) \Big/ 4\pi k$$

where r_1 = inner radius
r_{N+1} = outer radius

Table 43.6 Thermal Properties of Liquid Metals[a]

Composition	Melting Point (K)	T (K)	ρ (kg/m^3)	C_p (kJ/kg · K)	$\nu \times 10^7$ (m^2/sec)	k (W/m · K)	$\alpha \times 10^5$ (m^2/sec)	Pr
Bismuth	544	589	10,011	0.1444	1.617	16.4	0.138	0.0142
		1033	9,467	0.1645	0.8343	15.6	1.001	0.0083
Lead	600	644	10,540	0.159	2.276	16.1	1.084	0.024
		755	10,412	0.155	1.849	15.6	1.223	0.017
Mercury	234	273	13,595	0.140	1.240	8.180	0.429	0.0290
		600	12,809	0.136	0.711	11.95	0.688	0.0103
Potassium	337	422	807.3	0.80	4.608	45.0	6.99	0.0066
		977	674.4	0.75	1.905	33.1	6.55	0.0029
Sodium	371	366	929.1	1.38	7.516	86.2	6.71	0.011
		977	778.5	1.26	2.285	59.7	6.12	0.0037
NaK (56%/44%)	292	366	887.4	1.130	6.522	25.6	2.55	0.026
		977	740.1	1.043	2.174	28.9	3.74	0.0058
PbBi (44.5%/55.5%)	398	422	10,524	0.147	—	9.05	0.586	—
		644	10,236	0.147	1.496	11.86	0.790	0.189

[a]Adapted from *Liquid Metals Handbook*, The Atomic Energy Commission, Department of the Navy, Washington, DC, 1952.

Table 43.7 Thermal Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m^3)	C_p (kJ/kg · K)	$\nu \times 10^6$ (m^2/sec)	k (W/m · K)	$\alpha \times 10^4$ (m^2/sec)	Pr
Air						
100	3.6010	1.0266	1.923	0.009246	0.0250	0.768
300	1.1774	1.0057	16.84	0.02624	0.2216	0.708
2500	0.1394	1.688	543.0	0.175	7.437	0.730
Ammonia, Nh$_3$						
220	0.3828	2.198	19.0	0.0171	0.2054	0.93
473	0.4405	2.395	37.4	0.0467	0.4421	0.84
Carbon Dioxide						
220	2.4733	0.783	4.490	0.01081	0.0592	0.818
600	0.8938	1.076	30.02	0.04311	0.4483	0.668
Carbon Monoxide						
220	1.5536	1.0429	8.903	0.01906	0.1176	0.758
600	0.5685	1.0877	52.06	0.04446	0.7190	0.724
Helium						
33	1.4657	5.200	3.42	0.0353	0.04625	0.74
900	0.05286	5.200	781.3	0.298	10.834	0.72
Hydrogen						
30	0.8472	10.840	1.895	0.0228	0.02493	0.759
300	0.0819	14.314	109.5	0.182	1.554	0.706
1000	0.0819	14.314	109.5	0.182	1.554	0.706
Nitrogen						
100	3.4808	1.0722	1.971	0.009450	0.02531	0.786
300	1.1421	1.0408	15.63	0.0262	0.204	0.713
1200	0.2851	1.2037	156.1	0.07184	2.0932	0.748
Oxygen						
100	3.9918	0.9479	1.946	0.00903	0.02388	0.815
300	1.3007	0.9203	15.86	0.02676	0.2235	0.709
600	0.6504	1.0044	52.15	0.04832	0.7399	0.704
Steam (H_2O Vapor)						
380	0.5863	2.060	21.6	0.0246	0.2036	1.060
850	0.2579	2.186	115.2	0.0637	1.130	1.019

[a]Adapted from Ref. 2.

43.2.3 Two-Dimensional Steady-State Heat Conduction

Two-dimensional heat transfer in an isotropic, homogeneous material with no internal heat generation requires solution of the heat diffusion equation of the form $\partial^2T/\partial X^2 + \partial T/\partial y^2 = 0$, referred to as the *Laplace equation.* For certain geometries and a limited number of fairly simple combinations of boundary conditions, exact solutions can be obtained analytically. However, for anything but simple geometries or for simple geometries with complicated boundary conditions, development of an appropriate analytical solution can be difficult and other methods are usually employed. Among these are solution procedures involving the use of *graphical* or *numerical* approaches. In the first of these, the rate of heat transfer between two isotherms T_1 and T_2 is expressed in terms of the conduction shape factor, defined by

$$q = kS(T_1 - T_2)$$

Table 43.9 illustrates the shape factor for a number of common geometric configurations. By combining these shape factors, the heat transfer characteristics for a wide variety of geometric configurations can be obtained.

Prior to the development of high-speed digital computers, shape factor and analytical methods were the most prevalent methods utilized for evaluating steady-state and transient conduction problems. However, more recently, solution procedures for problems involving complicated geometries

Table 43.8 One-Dimensional Heat Conduction

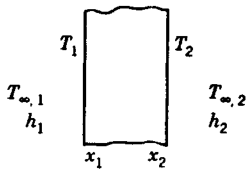
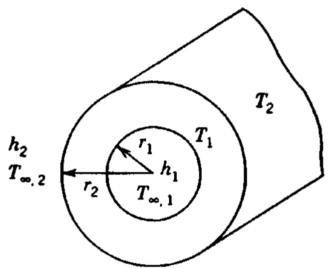
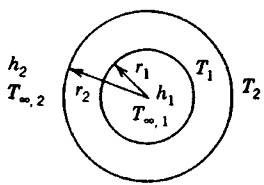
<table>
<tr><th>Geometry</th><th></th><th>Heat-Transfer Rate and Temperature Distribution</th><th>Heat-Transfer Rate and Overall Heat-Transfer Coefficient with Convection at the Boundaries</th></tr>
<tr><td>Plane wall</td><td>
</td><td>$q = \dfrac{T_1 - T_2}{(x_2 - x_1)/kA}$
$T = T_1 + \dfrac{T_2 - T_1}{x_x - x_1}(x - x_1)$
$R = (x_x - x_1)/kA$</td><td>$q = UA(T_{\infty,1} - T_{\infty,2})$
$U = \dfrac{1}{\dfrac{1}{h_1} + \dfrac{x_2 - x_2}{k} + \dfrac{11}{h_2}}$</td></tr>
<tr><td>Hollow cylinder</td><td>
</td><td>$q = \dfrac{T_1 - T_2}{[\ln(r_2/r_1)]/2\pi kL}$
$T = \dfrac{T_2 - T_1}{\ln(r_2/r_1)} \ln \dfrac{r}{r_1}$
$R = \dfrac{\ln(r_2/r_1)}{2\pi kL}$</td><td>$q = 2\pi r_1 L U_1 (T_{\infty,1} - T_{\infty,2})$
$= 2\pi r_1 L U_2 (T_{\infty,1} - T_{\infty,2})$
$U_1 = \dfrac{1}{\dfrac{1}{h_1} + \dfrac{r_1 \ln(r_2/r_1)}{k} + \dfrac{r_1}{r_2}\dfrac{1}{h_2}}$
$U_2 = \dfrac{1}{\left(\dfrac{r_2}{r_1}\right)\dfrac{1}{h_1} + \dfrac{r_2 \ln(r_2/r_1)}{k} + \dfrac{1}{h_2}}$</td></tr>
<tr><td>Hollow sphere</td><td>
</td><td>$q = \dfrac{T_2 - T_2}{\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)\Big/ 4\pi k}$
$T = \dfrac{1}{\left(1 - \dfrac{r_1}{r_2}\right)}\left[\dfrac{r_1}{r}(T_1 - T_2) + \left(T_2 - T_1\dfrac{r_1}{r_2}\right)\right]$
$R = \left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)\Big/ 4\pi k$</td><td>$q = 4\pi r_1^2 U_1 (T_{\infty,1} - T_{\infty,2})$
$= 4\pi r_2^2 U_2 (T_{\infty,1} - T_{\infty,2})$
$U_1 = \dfrac{1}{\dfrac{1}{h_1} + r_1^2\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)\Big/ k + \left(\dfrac{r_1}{r_2}\right)^2 \dfrac{1}{h_2}}$
$U_2 = \dfrac{1}{\left(\dfrac{r_1}{r_2}\right)^2 \dfrac{1}{h_1} + r_2^2\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)\Big/ k + \dfrac{1}{h_2}}$</td></tr>
</table>

Table 43.9 Conduction Shape Factors

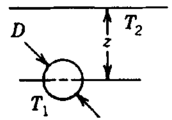
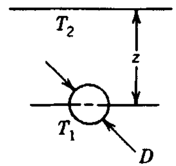
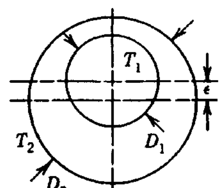
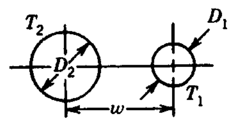
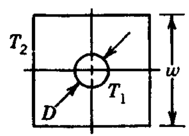
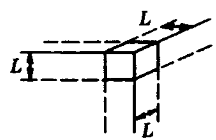
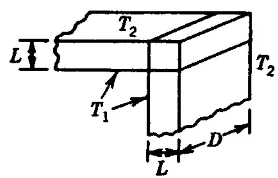
System	Schematic	Restrictions	Shape Factor
Isothermal sphere buried in a semi-infinite medium having isothermal surface		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Horizontal isothermal cylinder of length L buried in a semi-infinite medium having isothermal surface		$L >> D$ $\left.\begin{matrix} L >> D \\ z > 3D/2 \end{matrix}\right\}$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$ $\dfrac{2\pi L}{\ln(4z/D)}$
The cylinder of length L with eccentric bore		$L >> D_1, D_2$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4\varepsilon^2}{2D_1D_2}\right)}$

Table 43.9 ***(Continued)***

System	Schematic	Restrictions	Shape Factor
Conduction between two cylinders of length L in infinite medium		$L >> D_1, D_2$ $L >> W$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4W^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$
Circular cylinder of length L in a square solid		$w > D$	$\dfrac{2\pi L}{\ln(1.08\ w/D)}$
Conduction through the edge of adjoining walls		$D > L/5$	$0.54D$
Conduction through corner of three walls with inside and outside temperature, respectively, at T_1 and T_2		$L <<$ length and width of wall	$0.15L$

or boundary conditions have utilized the finite difference method (FDM). In this method, the solid object is divided into a number of distinct or discrete regions, referred to as *nodes,* each with a specified boundary condition. An energy balance is then written for each nodal region and these equations are solved simultaneously. For interior nodes in a two-dimensional system with no internal heat generation, the energy equation takes the form of the Laplace equation, discussed earlier. However, because the system is characterized in terms of a nodal network, a finite difference approximation must be used. This approximation is derived by substituting the following equation for the x-direction rate of change expression

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta x)^2}$$

and for the y-direction rate of change expression

$$\left.\frac{\partial^2 T}{\partial y^2}\right|_{m,n} \frac{T_{m,n+1} + T_{m,n-1} + T_{m,n}}{(\Delta y)^2}$$

Assuming $\Delta x = \Delta y$ and substituting into the Laplace equation results in the following expression:

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0$$

which reduces the exact difference equation to an approximate algebraic expression.

Combining this temperature difference with Fourier's law yields an expression for each internal node:

$$T_{m,n+1} + T_{m,n+1} + T_{m-1,n} + T_{m-1,n} + \frac{\dot{q}\Delta x \cdot \Delta y \cdot 1}{k} - 4T_{m,n} = 0$$

Similar equations for other geometries (i.e., corners) and boundary conditions (i.e., convection) and combinations of the two are listed in Table 43.10. These equations must then be solved using some form of matrix inversion technique, Gauss–Seidel iteration method, or other method for solving large numbers of simultaneous equations.

43.2.4 Heat Conduction with Convection Heat Transfer on the Boundaries

In physical situations where a solid is immersed in a fluid, or a portion of the surface is exposed to a liquid or gas, heat transfer will occur by convection (or when there is a large temperature difference, through some combination of convection and/or radiation). In these situations, the heat transfer is governed by *Newton's law of cooling,* which is expressed as

$$q = hA\Delta T$$

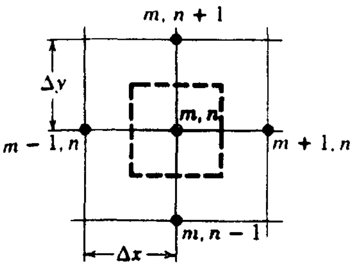
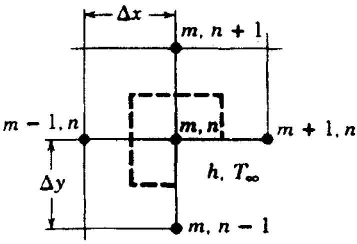
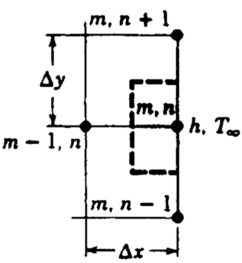
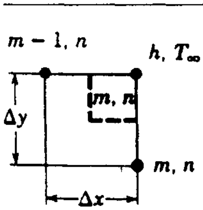
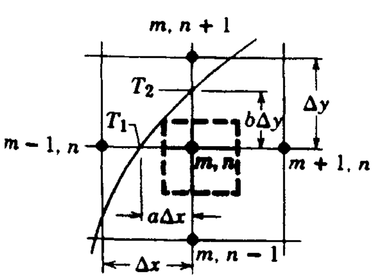
where h is the *convection heat transfer coefficient* (Section 43.2), ΔT is the temperature difference between the solid surface and the fluid, and A is the surface area in contact with the fluid. The resistance occurring at the surface abounding the solid and fluid is referred to as the *thermal resistance* and is given by $1/hA$, i.e., the *convection resistance.* Combining this resistance term with the appropriate conduction resistance yields an *overall heat transfer coefficient* U. Usage of this term allows the overall heat transfer to be defined as $q = UA\Delta T$.

Table 43.8 shows the overall heat transfer coefficients for some simple geometries. Note that U may be based either on the inner surface (U_1) or on the outer surface (U_2) for the cylinders and spheres.

Critical Radius of Insulation for Cylinders

A large number of practical applications involve the use of insulation materials to reduce the transfer of heat to or from cylindrical surfaces. This is particularly true of steam or hot water pipes, where concentric cylinders of insulation are typically added to the outside of the pipes to reduce the heat loss. Beyond a certain thickness, however, the continued addition of insulation may not result in continued reductions in the heat loss. To optimize the thickness of insulation required for these types of applications, a value typically referred to as the *critical radius,* defined as $r_{cr} = k/h$, is used. If the outer radius of the object to be insulated is less than r_{cr}, then the addition of insulation will increase the heat loss, while for cases where the outer radii is greater than r_{cr}, any additional increases in insulation thickness will result in a decrease in heat loss.

Table 43.10 Summary of Nodal Finite-Difference Equations

Configuration	Finite-Difference Equation for $\Delta x = \Delta y$
	$T_{m,n+1} + T_{m,n-1} + T_{m-1,n}$ $-4T_{m,n} = 0$ **Case 1.** Interior node.
	$2(T_{m-1,n} + T_{m,n+1}) + (T_{m+1,n} + T_{m,n-1}) + 2\frac{h\Delta x}{k}T_\infty - 2\left(3 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$ **Case 2.** Node at an internal corner with convection.
	$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 2\right)T_{m,n} = 0$ **Case 3.** Node at a plane surface with convection.
	$(T_{m,n-1}$ pl $T_{m-1,n}) + 2\frac{h\Delta x}{k}T_\infty$ $-2\left(\frac{h\Delta x}{k} + 1\right)T_{m,n} = 0$ **Case 4.** Node at an external corner with convection.
	$\frac{2}{a+1}T_{m+1,n} + \frac{2}{b+1}T_{m,n-1} + \frac{2}{a(a+1)}T_1 + \frac{2}{b(b+1)}T_2 - \left(\frac{2}{a} + \frac{2}{b}\right)T_{m,n} = 0$ **Case 5.** Node near a curved surface maintained at a nonuniform temperature.

Extended Surfaces

In examining Newton's law of cooling, it is clear that the rate of heat transfer between a solid and the surrounding ambient fluid may be increased by increasing the surface area of the solid that is exposed to the fluid. This is typically done through the addition of extended surfaces or fins to the primary surface. Numerous examples exist, including the cooling fins on air-cooled engines, such as motorcycles or lawn mowers, or the fins attached to automobile radiators.

Figure 43.2 illustrates a common uniform cross-section extended surface, fin, with a constant base temperature T_b, a constant cross-sectional area A, a circumference of $C = 2W + 2t$, and a length L that is much larger than the thickness t. For these conditions, the temperature distribution in the fin must satisfy the following expression:

$$\frac{d^2T}{dx^2} - \frac{hC}{kA}(T - T_\infty) = 0$$

The solution of this equation depends upon the boundary conditions existing at the tip, that is, at $x = L$. Table 43.11 shows the temperature distribution and heat transfer rate for fins of uniform cross section subjected to a number of different tip conditions, assuming a constant value for the heat transfer coefficient h.

Two terms are used to evaluate fins and their usefulness. *Fin effectiveness* is defined as the ratio of heat transfer rate with the fin to the heat transfer rate that would exist if the fin were not used. For most practical applications, the use of a fin is justified only when the fin effectiveness is significantly greater than 2. *Fin efficiency* η_f represents the ratio of the actual heat transfer rate from a fin to the heat transfer rate that would occur if the entire fin surface could be maintained at a uniform temperature equal to the temperature of the base of the fin. For this case, Newton's law of cooling can be written as

$$q = \eta_f h A_f (T_b - T_\infty)$$

where A_f is the total surface area of the fin and T_b is the temperature of the fin at the base. The application of fins for heat removal can be applied to either forced or natural convection of gases, and while some advantages can be gained in terms of increasing the liquid–solid or solid–vapor surface area, fins as such are not normally utilized for situations involving phase change heat transfer, such as boiling or condensation.

43.2.5 Transient Heat Conduction

If a solid body, all at some uniform temperature $T_{\infty i}$, is immersed in a fluid of different temperature T_∞, the surface of the solid body may be subject to heat losses (or gains) through convection from

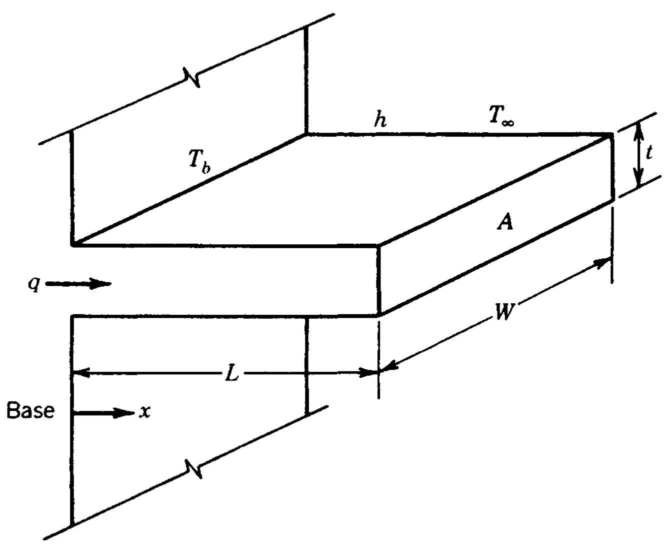


Fig. 43.2 Heat transfer by extended surfaces.

Table 43.11 Temperature Distribution and Heat Transfer Rate at the Fin Base ($m = \sqrt{hc/kA}$)

Condition at $x = L$	$\dfrac{T - T_\infty}{T_b - T_\infty}$	Heat Transfer Rate $q/mKA\,(T_b - T_\infty)$
$h(T_{x=L} - T_\infty) = -k\left(\dfrac{dT}{dx}\right)_{x=L}$ (convection)	$\dfrac{\cosh m(L - x) + \dfrac{h}{mk}\sinh m(L - x)}{\cosh mL + \dfrac{h}{mk}\sinh mL}$	$\dfrac{\sinh mL + \dfrac{h}{mk}\cosh mL}{\cosh mL + \dfrac{h}{mk}\sinh mL}$
$\left(\dfrac{dT}{dx}\right)_{x=L} = 0$ (insulated)	$\dfrac{\cosh m(L - x)}{\cosh mL}$	$\tanh mL$
$T_{x=L} = T_L$ (prescribed temperature)	$\dfrac{(T_L - T_\infty)/(T_b - T_\infty)\sinh mx + \sinh m(L - x)}{\sinh ml}$	$\dfrac{\cosh mL - (T_L - T_\infty)/(T_b - T_\infty)}{\sinh ml}$
$T_{x=L} = T_\infty$ (infinitely long fin, $L \rightarrow \infty$)	e^{-mx}	1

the surface. In this situation, the heat lost (or gained) at the surface results from the conduction of heat from inside the body. To determine the significance of these two heat transfer modes, a dimensionless parameter referred to as the *Biot number* is used. This dimensionless number, defined as $\mathrm{Bi} = hL/k$ where $L = V/A$ or the ratio of the volume of the solid to the surface area of the solid, really represents a comparative relationship of the importance of convection from the outer surface to the conduction occurring inside. When this value is less than 0.1, the temperature of the solid may be assumed uniform and dependent on time alone. When this value is greater than 0.1, there is some spatial temperature variation that will affect the solution procedure.

For the first case, that is, $\mathrm{Bi} < 0.1$, an approximation referred to as the *lumped heat-capacity* method may be used. In this method, the temperature of the solid is given by the expression

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(\frac{-t}{\tau_t}\right) = \exp\left(-\mathrm{BiFo}\right)$$

where τ_t is the *time constant* and is equal to $\rho C_p V/hA$. Increasing the value of the time constant, τ_t, will result in a decrease in the thermal response of the solid to the environment and hence will increase the time required to reach thermal equilibrium (i.e., $T = T_\infty$). In this expression, Fo represents the dimensionless time and is called the *Fourier number*, the value of which is equal to $\alpha t A^2/V^2$. The Fourier number, along with the Biot number, can be used to characterize transient heat conduction problems. The total heat flow through the surface of the solid over the time interval from $t = 0$ to time t can be expressed as

$$Q = \rho V C_p(T_i - T\infty)[1 - \exp(-t/\tau_t]$$

Transient Heat Transfer for Infinite Plate, Infinite Cylinder, and Sphere Subjected to Surface Convection

Generalized analytical solutions to transient heat transfer problems involving infinite plates, cylinders, and finite diameter spheres subjected to surface convection have been developed. These solutions can be presented in graphical form through the use of the *Heisler charts*,[3] illustrated in Figs. 43.3–43.11 for plane walls, cylinders, and spheres, respectively. In this procedure, the solid is assumed to be at a uniform temperature T_i at time $t = 0$ and then is suddenly subjected or immersed in a fluid at a uniform temperature T_∞. The convection heat-transfer coefficient h is assumed to be constant, as is the temperature of the fluid. Combining Figs. 43.3 and 43.4 for plane walls; Figs. 43.6 and 43.7 for cylinders; Figs. 43.9 and 43.10 for spheres, allows the resulting time-dependent temperature of any point within the solid to be found. The total amount of energy Q transferred to or from the solid surface from time $t = 0$ to time t can be found from Figs. 43.5, 43.8, and 43.11.

43.3 CONVECTION HEAT TRANSFER

As discussed earlier, convection heat transfer is the mode of energy transport in which the energy is transferred by means of fluid motion. This transfer can be the result of the random molecular motion or bulk motion of the fluid. If the fluid motion is caused by external forces, the energy transfer is called *forced convection*. If the fluid motion arises from a buoyancy effect caused by density differences, the energy transfer is called *free convection* or *natural convection*. For either case, the heat-transfer rate, q, can be expressed in terms of the surface area, A, and the temperature difference, ΔT, by Newton's law of cooling:

$$q = hA\Delta T$$

In this expression, h is referred to as the convection heat-transfer coefficient or film coefficient, which is a function of the velocity and physical properties of the fluid and the shape and nature of the surface. The nondimensional heat-transfer coefficient $\mathrm{Nu} = hL/k$ is called the *Nusselt number*, where L is a characteristic length and k is the thermal conductivity of the fluid.

43.3.1 Forced Convection—Internal Flow

For internal flow in a tube or pipe, the convection heat-transfer coefficient is typically defined as a function of the temperature difference existing between the temperature at the surface of the tube and the *bulk* or *mixing-cup temperature* T_b, that is, $\Delta T = T_s - T_b$, which can be defined as

$$T_b = \frac{\int C_p T dm}{\int C_p dm}$$

where m is the axial flow rate. Using this value, the heat transfer between the tube and the fluid can be written as $q = hA(T_s - T_b)$.

In the entrance region of a tube or pipe, the flow is quite different from that occurring downstream from the entrance. The rate of heat transfer differs significantly depending on whether the flow is

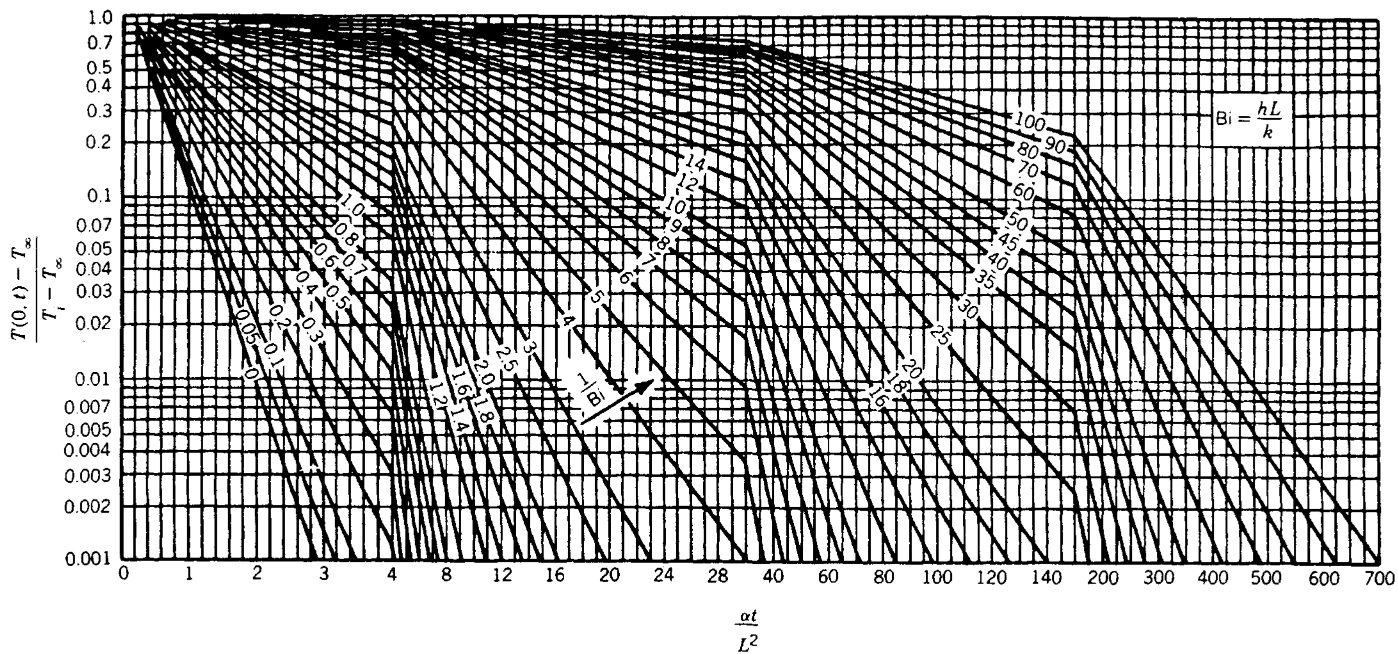


Fig. 43.3 Midplane temperature as a function of time for a plane wall of thickness 2*L*. (Adapted from Heisler.[3])

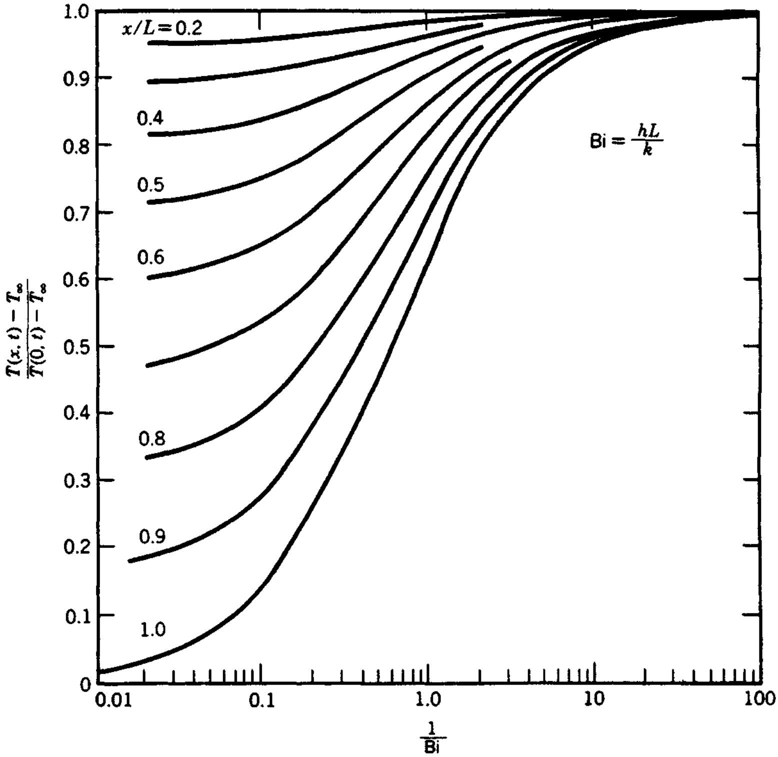


Fig. 43.4 Temperature distribution in a plane wall of thickness 2*L*. (Adapted from Heisler.[3])

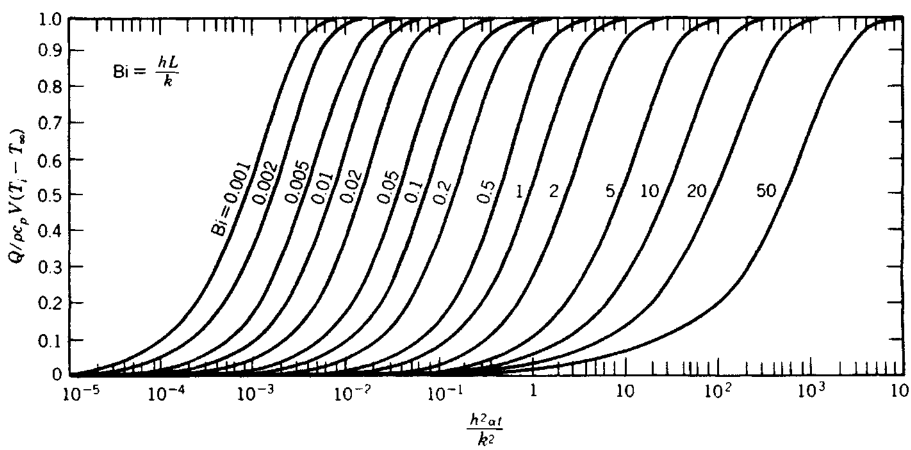


Fig. 43.5 Internal energy change as a function of time for a plane wall of thickness 2*L*.[4] (Used with the permission of McGraw-Hill Book Company.)

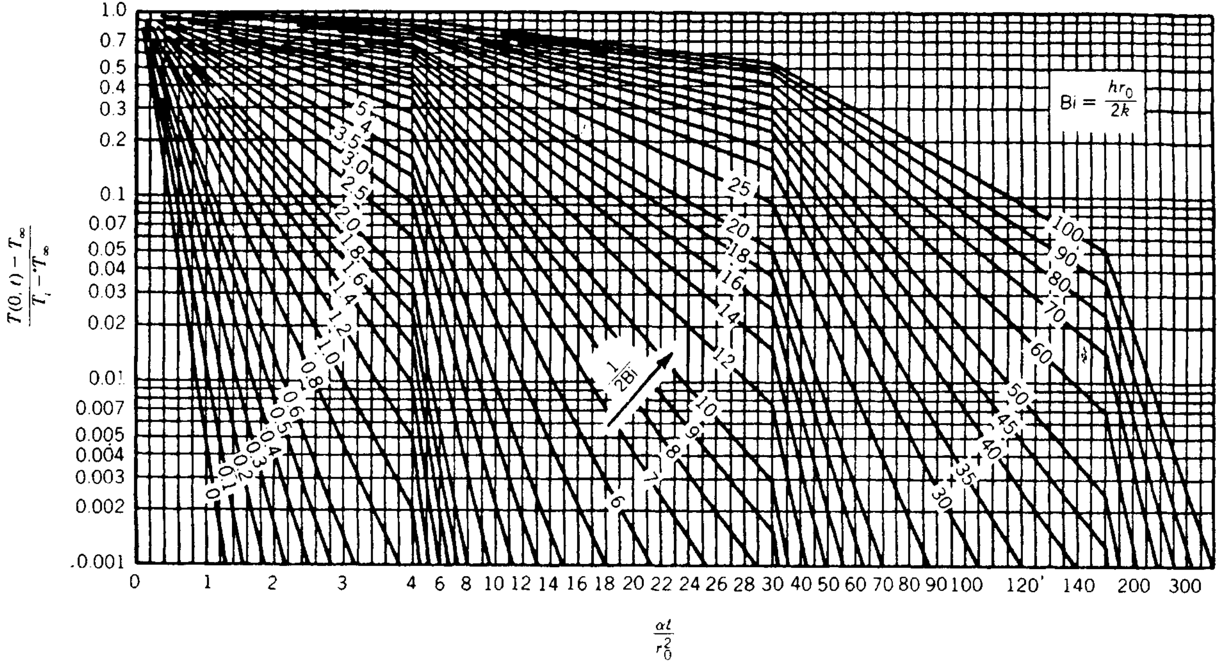


Fig. 43.6 Centerline temperature as a function of time for an infinite cylinder of radius r_o. (Adapted from Heisler.[3])

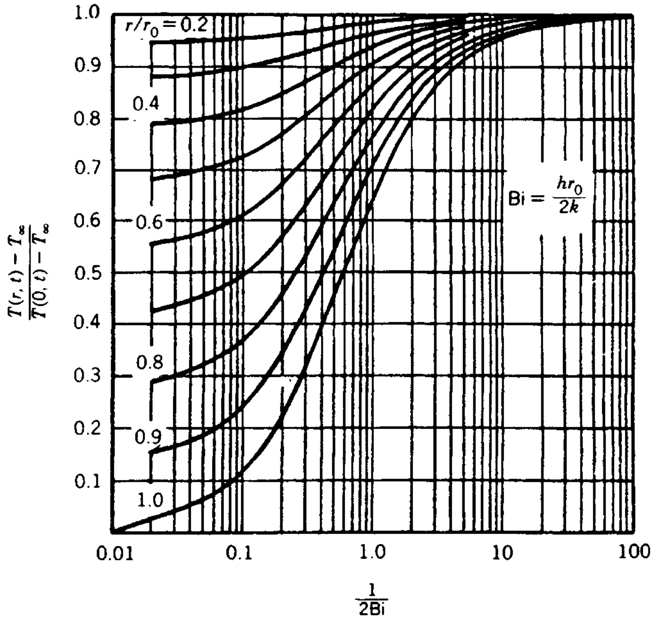


Fig. 43.7 Temperature distribution in an infinite cylinder of radius r_o. (Adapted from Heisler.[3])

laminar or *turbulent*. From fluid mechanics, the flow is considered to be turbulent when $\mathrm{Re}_D = V_m D/v > 2300$ for a smooth tube. This transition from laminar to turbulent, however, also depends on the roughness of tube wall and other factors. The generally accepted range for transition is $2000 < \mathrm{Re}_D < 4000$.

Laminar Fully Developed Flow

For situations where both the thermal and velocity profiles are fully developed, the Nusselt number is constant and depends only on the thermal boundary conditions. For *circular tubes* with $\mathrm{Pr} \geq 0.6$ and $x/D\mathrm{Re}_D\ \mathrm{Pr} > 0.05$, the Nusselt numbers have been shown to be $\mathrm{Nu}_D = 3.66$ and 4.36 for constant temperature and constant heat flux conditions, respectively. Here, the fluid properties are based on the mean bulk temperature.

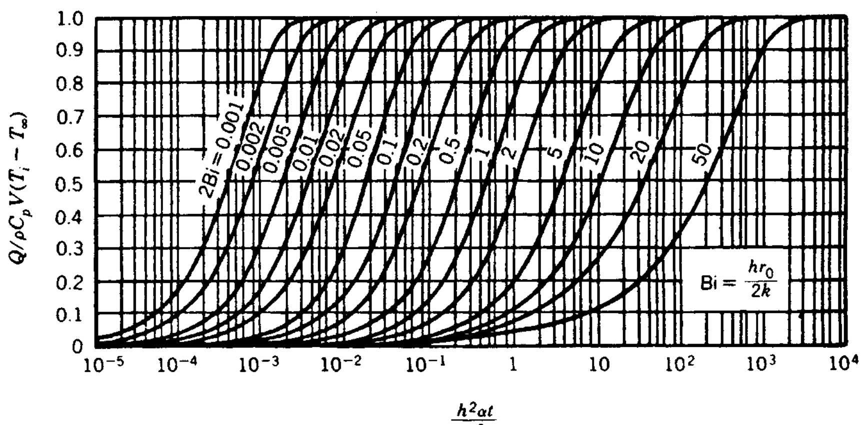


Fig. 43.8 Internal energy change as a function of time for an infinite cylinder of radius r_o.[4] (Used with the permission of McGraw-Hill Book Company.)

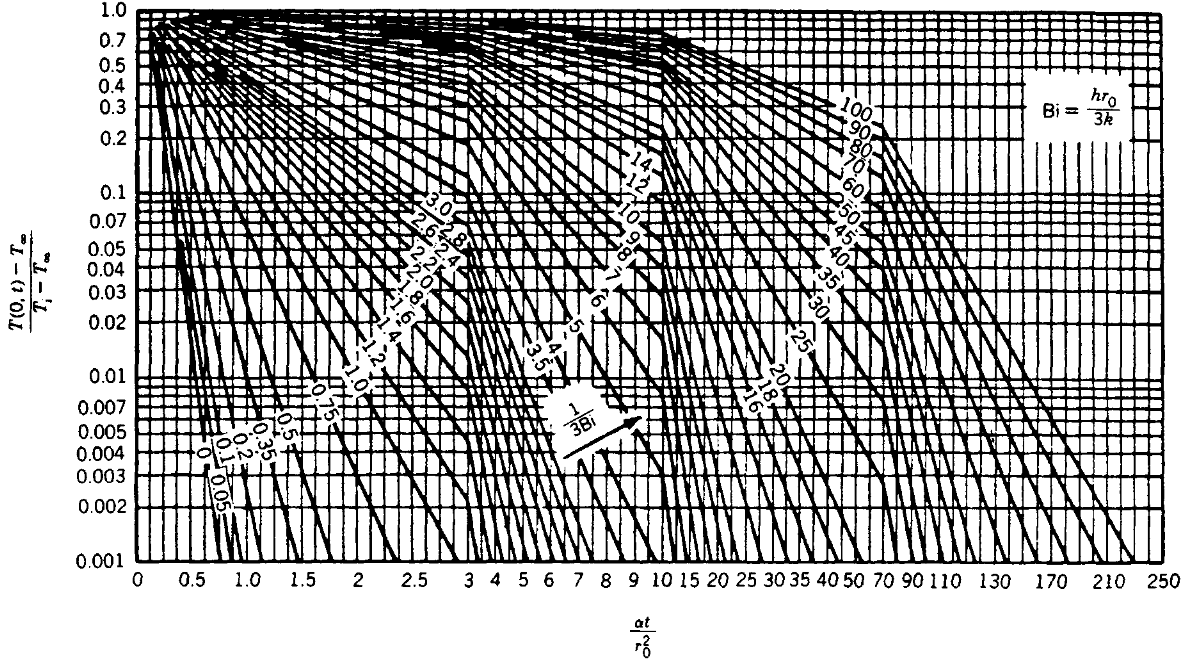


Fig. 43.9 Center temperature as a function of time in a sphere of radius r_o. (Adapted from Heisler.[3])

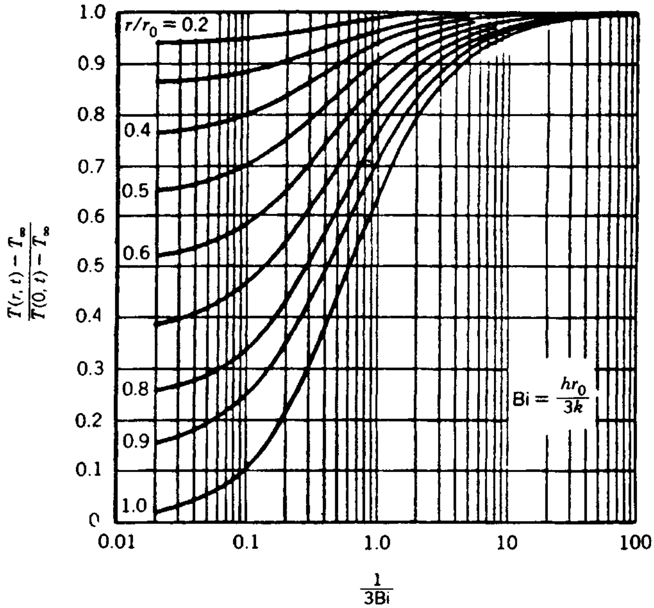


Fig. 43.10 Temperature distribution in a sphere of radius r_o. (Adapted from Heisler.[3])

For *noncircular tubes,* the hydraulic diameter, $D_h = 4 \times$ the flow cross-sectional area/wetted perimeter, is used to define the Nusselt number Nu_D and the Reynolds number Re_D. Table 43.12 shows the Nusselt numbers based on hydraulic diameter for various cross-sectional shapes.

Laminar Flow for Short Tubes

At the entrance of a tube, the Nusselt number is infinite, and decreases asymptotically to the value for fully developed flow as the flow progresses down the tube. The Sieder–Tate equation[5] gives good correlation for the combined entry length, that is, that region where the thermal and velocity profiles are both developing or for short tubes:

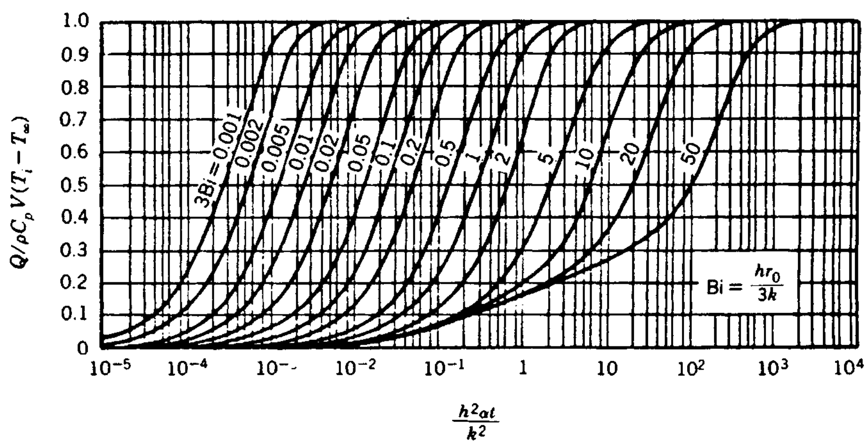
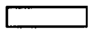
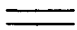
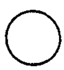


Fig. 43.11 Internal energy change as a function of time for a sphere of radius r_o.[4] (Used with the permission of McGraw-Hill Book Company.)

Table 43.12 Nussselt Numbers for Fully Developed Laminar Flow for Tubes of Various Cross Sections[a]

Geometry ($L/DH > 100$)	Nu_{H1}	Nu_{H2}	Nu_r
2b  2a, $\frac{2b}{2a} = 1$	3.608	3.091	2.976
2b  2a, $\frac{2b}{2a} = \frac{1}{2}$	4.123	3.017	3.391
2b  2a, $\frac{2b}{2a} = \frac{1}{4}$	5.099	4.35	3.66
2b  2a, $\frac{2b}{2a} = \frac{1}{8}$	6.490	2.904	5.597
 $\frac{2b}{2a} = 0$	8.235	8.235	7.541
$\frac{b}{a} = 0$	5.385	—	4.861
	4.364	4.364	3.657

[a]Nu_{H1} = average Nusselt number for uniform heat flux in flow direction and uniform wall temperature at paticular flow cross section.
Nu_{H2} = average Nusselt number for uniform heat flux both in flow direction and around periphery.
Nu_{Hrr} = average Nusselt number for uniform wall temperature.

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 1.86(Re_D Pr)^{1/3} \left(\frac{D}{L}\right)^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

for T_s = constant, $0.48 < Pr < 16{,}700$, $0.0044 < \mu/\mu_s < 9.75$, and $(Re_D\, Pr\, D/L)^{1/3} (\mu/\mu_s)^{0.14} > 2$.

In this expression, all of the fluid properties are evaluated at the mean bulk temperature except for μ_s which is evaluated at the wall surface temperature. The average convection heat-transfer coefficient $\bar{h}$ is based on the arithmetic average of the inlet and outlet temperature differences.

Turbulent Flow in Circular Tubes

In turbulent flow, the velocity and thermal entry lengths are much shorter than for a laminar flow. As a result, with the exception of short tubes, the fully developed flow values of the Nusselt number are frequently used directly in the calculation of the heat transfer. In general, the Nusselt number obtained for the constant heat flux case is greater than the Nusselt number obtained for the constant temperature case. The one exception to this is the case of liquid metals, where the difference is smaller than for laminar flow and becomes negligible for $Pr > 1.0$. The Dittus–Boelter equation[6] is typically used if the difference between the pipe surface temperature and the bulk fluid temperature is less than 6°C (10°F) for liquids or 56°C (100°F) for gases:

$$Nu_D = 0.023\, Re_D^{0.8}\, Pr^n$$

for $0.7 \le Pr \le 160$, $Re_D \ge 10{,}000$ and $L/D \ge 60$, where

$$n = 0.4 \text{ for heating, } T_s > T_b$$
$$= 0.3 \text{ for cooling, } T_s < T_b$$

For temperature differences greater than those specified above, use[5]

$$\mathrm{Nu}_D = 0.027\mathrm{Re}_D^{0.8}\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

for $0.7 \leq \mathrm{Pr} \leq 16{,}700$, $\mathrm{Re}_D \geq 10{,}000$ and $L/D \geq 60$.

In this expression, the properties are all evaluated at the mean bulk fluid temperature, with the exception of μ_s, which is again evaluated at the tube surface temperature.

For *concentric tube annuli,* the hydraulic diameter $D_h = D_o - D_i$ (outer diameter–inner diameter) must be used for Nu_D and Re_D, and the coefficient h at either surface of the annulus must be evaluated from the Dittus–Boelter equation. Here, it should be noted that the foregoing equations apply for smooth surfaces and that the heat-transfer rate will be larger for rough surfaces and are not applicable to liquid metals.

Fully Developed Turbulent Flow of Liquid Metals in Circular Tubes

Because the Prandtl number for liquid metals is on the order of 0.01, the Nusselt number is primarily dependent upon a dimensionless parameter number referred to as the *Peclet number,* which in general is defined as Pe = RePr:

$$\mathrm{Nu}_D = 5.0 + 0.025Pe_D^{0.8}$$

which is valid for situations where T_s = a constant and $\mathrm{Pe}_D > 100$ and $L/D > 60$.

For q'' = constant and $3.6 \times 10^3 < \mathrm{Re}_D < 9.05 \times 10^5$, $10^2 < \mathrm{Pe}_D < 10^4$, and $L/D > 60$, the Nusselt number can be expressed as

$$\mathrm{Nu}_D = 4.8 + 0.0185Pe_D^{0.827}$$

43.3.2 Forced Convection—External Flow

In forced convection heat transfer, the heat transfer coefficient h is based on the temperature difference between the wall surface temperature and the fluid temperature in the free stream outside the thermal boundary layer. The total heat-transfer rate from the wall to the fluid is given by $q = hA\ (T_s - T_\infty)$. The Reynolds numbers are based on the free stream velocity. The fluid properties are evaluated either at the free stream temperature T_∞ or at the film temperature $T_f = (T_s + T_\infty)/2$.

Laminar Flow on a Flat Plate

When the flow velocity along a constant temperature semi-infinite plate is uniform, the boundary layer originates from the leading edge and is laminar and the flow remains laminar until the local Reynolds number, $\mathrm{Re}_x = U_\infty x/v$, reaches the *critical Reynolds number* Re_c. When the surface is smooth, the Reynolds number is generally assumed to be $\mathrm{Re}_c = 5 \times 10^5$; however, the value will depend on several parameters, including the surface roughness.

For a given distance x from the leading edge, the *local Nusselt number* and the *average Nusselt number* between $x = 0$ and $x = L$ are given below (Re_x and $\mathrm{Re}_L \leq 5 \times 10^5$):

$$\left.\begin{aligned} \mathrm{Nu}_x &= hx/k = 0.332\mathrm{Re}_x^{0.5}\ \mathrm{Pr}^{1/3} \\ \overline{\mathrm{Nu}}_L &= \bar{h}L/k = 0.664\mathrm{Re}_L^{0.5}\ \mathrm{Pr}^{1/3} \end{aligned}\right\} \quad \text{for Pr} \geq 0.6$$

$$\left.\begin{aligned} \mathrm{Nu}_x &= 0.565(\mathrm{Re}_x\ \mathrm{Pr})^{0.5} \\ \overline{\mathrm{Nu}}_L &= 1.13(\mathrm{Re}_L\ \mathrm{Pr})^{0.5} \end{aligned}\right\} \quad \text{for Pr} \leq 0.6$$

Here, all of the fluid properties are evaluated at the mean or average film temperature.

Turbulent Flow on a Flat Plate

When the flow over a flat plate is turbulent from the leading edge, expressions for the local Nusselt number can be written as

$$\mathrm{Nu}_x = 0.0292\mathrm{Re}_x^{0.8}\ \mathrm{Pr}^{1/3}$$

$$\overline{\mathrm{Nu}}_L = 0.036\mathrm{Re}_L^{0.8}\ \mathrm{Pr}^{1/3}$$

where the fluid properties are all based on the mean film temperature and $5 \times 10^5 \leq \mathrm{Re}_x$ and $\mathrm{Re}_L \leq 10^8$ and $0.6 \leq \mathrm{Pr} \leq 60$.

The Average Nusselt Number Between $x = 0$ and $x = L$ with Transition

For situations where transition occurs immediately once the critical Reynolds number Re_c has been reached,[7]

$$\overline{\mathrm{Nu}}_L = 0.036\mathrm{Pr}^{1/3}[\mathrm{Re}_L^{0.8} - \mathrm{Re}_c^{0.8} + 18.44\mathrm{Re}_c^{0.5}]$$

provided that $5 \times 10^5 \leq \text{Re}_L \leq 10^8$ and $0.6 \leq \text{Pr} \leq 60$. Specialized cases exist for this situation, such as

$$\overline{\text{Nu}}_L = 0.036\text{Pr}^{1/3}(\text{Re}_L^{0.8} - 18{,}700)$$

for $\text{Re}_c = 4 \times 10^5$ or

$$\overline{\text{Nu}}_L = 0.036\text{Pr}^{1/3}(\text{Re}_L^{0.8} - 23{,}000)$$

for $\text{Re}_c = 5 \times 10^5$. Again, all fluid properties are evaluated at the mean film temperature.

Circular Cylinders in Cross Flow

For circular cylinders in cross flow, the Nusselt number is based upon the diameter and can be expressed as

$$\overline{\text{Nu}}_D = (0.4\text{Re}_D^{0.5} + 0.06\text{Re}^{2/3})\text{Pr}^{0.4}(\mu_\infty/\mu_s)^{0.25}$$

for $0.67 < \text{Pr} < 300$, $10 < \text{Re}_D < 10^5$, and $0.25 < 5.2$. Here, the fluid properties are evaluated at the free stream temperature, except μ_s, which is evaluated at the surface temperature.[8]

Cylinders of Noncircular Cross Section in Cross Flow of Gases

For noncircular cylinders in cross flow, the Nusselt number is again based upon the diameter, but is expressed as

$$\overline{\text{Nu}}_D = C(\text{Re}_D)^m\ \text{Pr}^{1/3}$$

where C and m are listed in Table 43.13, and the fluid properties are evaluated at the mean film temperature.[9]

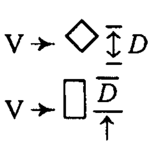
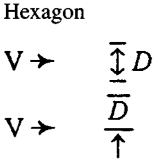
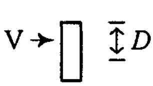
Flow Past a Sphere

For flow over a sphere, the Nusselt number is based upon the sphere diameter and can be expressed as

$$\overline{\text{Nu}}_D = 2 + (0.4\text{Re}_D^{0.5} + 0.006\text{Re}_D^{2/3})\text{Pr}^{0.4}(\mu_\infty/\mu_s)^{0.25}$$

for the case of $3.5 < \text{Re}_D < 8 \times 10^4$, $0.7 < \text{Pr} < 380$, and $1.0 < \mu_\infty/\mu_s < 3.2$. The fluid properties are calculated at the free stream temperature, except μ_s, which is evaluated at the surface temperature.[8]

Table 43.13 Constants and *m* for Noncircular Cylinders in Cross Flow

Geometry	Re_D	C	m
Square			
	5×10^3–10^5	0.246	0.588
	5×10^3–10^5	0.102	0.675
Hexagon			
	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
Vertical Plate			
	5×10^3–10^5	0.153	0.638
	4×10^3–1.5×10^4	0.228	0.721

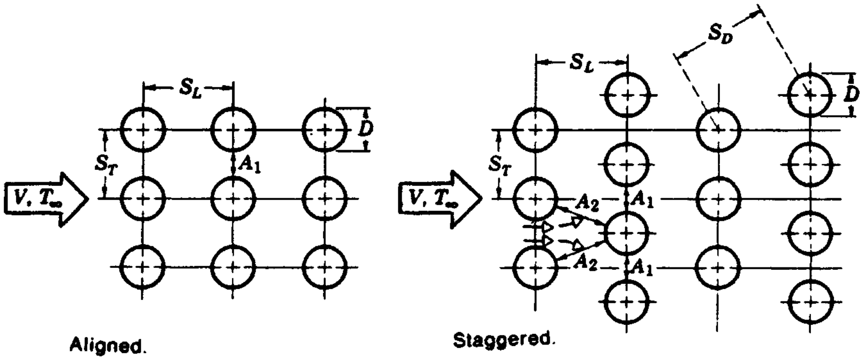


Fig. 43.12 Tube arrangement.

Flow across Banks of Tubes

For banks of tubes, the tube arrangement may be either *staggered* or *aligned* (Fig. 43.12), and the heat transfer coefficient for the first row is approximately equal to that for a single tube. In turbulent flow, the heat transfer coefficient for tubes in the first row is smaller than that of the subsequent rows. However, beyond the fourth or fifth row, the heat transfer coefficient becomes approximately constant. For tube banks with more than twenty rows, $0.7 < \mathrm{Pr} < 500$, and $1000 < \mathrm{Re}_{D,\max} < 2 \times 10^6$, the average Nusselt number for the entire tube bundle can be expressed as[10]

$$\overline{\mathrm{Nu}}_D = C(\mathrm{Re}_{D,\max})^m \,\mathrm{Pr}^{0.36}(\mathrm{Pr}_\infty/\mathrm{Pr}_s)^{0.25}$$

where all fluid properties are evaluated at T_∞ except Pr_s, which is evaluated at the surface temperature. The constants C and m used in this expression are listed in Table 43.14, and the Reynolds number is based on the maximum fluid velocity occurring at the minimum free flow area available for the fluid. Using the nomenclature shown in Fig. 43.12, the maximum fluid velocity can be determined by

$$V_{\max} = \frac{S_T}{S_T - D} V$$

for the aligned or staggered configuration provided

$$\sqrt{S_L^2 + (S_T/2)^2} > (S_T + D)/2$$

or as

$$V_{\max} = \frac{S_T}{\sqrt[2]{S_L^2 + (S_T/2)^2}} V$$

for staggered if

Table 43.14 Constants *C* and *m* of Heat-Transfer Coefficient for the Banks in Cross Flow

Configuration	$\mathrm{Re}_{D,\max}$	C	m
Aligned	10^3–2×10^5	0.27	0.63
Staggered ($S_T/S_L < 2$)	10^3–2×10^5	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered ($S_G/S_L > 2$)	10^3–2×10^5	0.40	0.60
Aligned	2×10^5–2×10^6	0.21	0.84
Staggered	2×10^5–2×10^6	0.022	0.84

$$\sqrt{S_L^2 + (S_T/2)^2} < (S_T + D)/2$$

Liquid Metals in Cross Flow over Banks of Tubes

The average Nusselt number for tubes in the inner rows can be expressed as

$$\overline{\mathrm{Nu}}_D = 4.03 + 0.228(\mathrm{Re}_{D,\max}\ \mathrm{Pr})^{0.67}$$

which is valid for $2 \times 10^4 < \mathrm{Re}_{D,\max} < 8 \times 10^4$ and $\mathrm{Pr} < 0.03$ and the fluid properties are evaluated at the mean film temperature.[11]

High-Speed Flow over a Flat Plate

When the free stream velocity is very high, the effects of viscous dissipation and fluid compressibility must be considered in the determination of the convection heat transfer. For these types of situations, the convection heat transfer can be described as $q = hA\ (T_s - T_{\alpha s})$, where $T_{\alpha s}$ is the *adiabatic surface temperature* or *recovery temperature,* and is related to the *recovery factor* by $r = (T_{\alpha s} - T_\infty)/(T_0 - T_\infty)$. The value of the *stagnation temperature* T_0 is related to the free stream static temperature T_∞ by the expression

$$\frac{T_0}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2$$

where γ is the specific heat ratio of the fluid and M_∞ is the ratio of the free stream velocity and the acoustic velocity. For the case where $0.6 < \mathrm{Pr} < 15$,

$$r = \mathrm{Pr}^{1/2} \quad \text{for laminar flow } (\mathrm{Re}_x < 5 \times 10^5)$$
$$r = \mathrm{Pr}^{1/3} \quad \text{for turbulent flow } (\mathrm{Re}_x > 5 \times 10^5)$$

Here, all of the fluid properties are evaluated at the reference temperature $T_{ref} = T_\infty + 0.5(T_s - T_\infty) + 0.22(T_{\alpha s} - T_\infty)$. Expressions for the local heat-transfer coefficients at a given distance x from the leading edge are given as:[2]

$$\mathrm{Nu}_x = 0.332\mathrm{Re}_x^{0.5}\ \mathrm{Pr}^{1/3} \qquad \text{for } \mathrm{Re}_x < 5 \times 10^5$$
$$\mathrm{Nu}_x = 0.0292\mathrm{Re}_x^{0.8}\ \mathrm{Pr}^{1/3} \qquad \text{for } 5 \times 10^5 < \mathrm{Re}_x < 10^7$$
$$\mathrm{Nu}_x = 0.185\mathrm{Re}_x(\log_{10}\mathrm{Re}_x)^{-2.584} \qquad \text{for } 10^7 < \mathrm{Re}_x < 10^9$$

In the case of gaseous fluids flowing at very high free stream velocities, dissociation of the gas may occur and will cause large variations in the properties within the boundary layer. For these cases, the heat-transfer coefficient must be defined in terms of the enthalpy difference, namely, $q = hA\ (i_s - i_{\alpha s})$, and the recovery factor will be given by $r = (i_s - i_{\alpha s})/(i_0 - i_\infty)$, where $i_{\alpha s}$ represents the enthalpy at the adiabatic wall conditions. Similar expressions to those shown above for Nu_x can be used by substituting the properties evaluated at a reference enthalpy defined as $i_{ref} = i_\infty + 0.5(i_s - i_\infty) + 0.22(i_{g}a_s - i_\infty)$.

High-Speed Gas Flow Past Cones

For the case of high-speed gaseous flows over conical-shaped objects, the following expressions can be used:

$$\mathrm{Nu}_x = 0.575\mathrm{Re}_x^{0.5}\ \mathrm{Pr}^{1/3} \qquad \text{for } \mathrm{Re}_x < 10^5$$
$$\mathrm{Nu}_x = 0.0292\mathrm{Re}_x^{0.8}\ \mathrm{Pr}^{1/3} \qquad \text{for } \mathrm{Re}_x > 10^5$$

where the fluid properties are evaluated at T_{ref}, as in the plate.[12]

Stagnation Point Heating for Gases

When the conditions are such that the flow can be assumed to behave as *incompressible,* the Reynolds number is based on the free stream velocity and $\bar{h}$ is defined as $q = \bar{h}A(T_s - T_\infty)$.[13] Estimations of the Nusselt number can be made using the following relationship:

$$\mathrm{Nu}_D = C\mathrm{Re}_D^{0.5}\ \mathrm{Pr}^{0.4}$$

where $C = 1.14$ for cylinders and 1.32 for spheres, and the fluid properties are evaluated at the mean film temperature. When the flow becomes *supersonic,* a bow shock wave will occur just off the front of the body. In this situation, the fluid properties must be evaluated at the stagnation state occurring behind the bow shock and the Nusselt number can be written as

$$\overline{\text{Nu}}_D = C\text{Re}_D^{0.5}\,\text{Pr}^{0.5}(\rho_\infty/\rho_0)^{0.25}$$

where $C = 0.95$ for cylinders and 1.28 for spheres, ρ_∞ is the free stream gas density, and ρ_o is the stagnation density of the stream behind the bow shock. The heat-transfer rate for this case is given by $q = \bar{h}A(T_s - T_0)$.

43.3.3 Free Convection

In free convection, the fluid motion is caused by the buoyant force resulting from the density difference near the body surface, which is at a temperature different from that of the free fluid far removed from the surface, where the velocity is zero. In all free convection correlations, except for the enclosed cavities, the fluid properties are usually evaluated at the mean film temperature $T_f = (T_1 + T_\infty)/2$. The thermal expansion coefficient β, however, is evaluated at the free fluid temperature T_∞. The convection heat transfer coefficient h is based on the temperature difference between the surface and the free fluid.

Free Convection from Flat Plates and Cylinders

For free convection from flat plates and cylinders, the average Nusselt number $\overline{\text{Nu}}_L$ can be expressed as[4]

$$\overline{\text{Nu}}_L = C(\text{Gr}_L\,\text{Pr})^m$$

where the constants C and m are given as shown in Table 43.15. The *Grashof Prandtal number* product, (Gr_LPr) is called the *Rayleigh number* (Ra_L) and for certain ranges of this value, Figs. 43.13 and 43.14 are used instead of the above equation. Reasonable approximations for other types of *three-dimensional shapes,* such as short cylinders and blocks, can be made for $10^4 < \text{Ra}_L < 10^9$, by using this expression and $C = 0.6$, $m = 1/4$, provided that the characteristic length, L, is determined from $1/L = 1/L_{hor} + 1/L_{ver}$, where L_{ver} is the height and L_{hor} is the horizontal dimension of the object in question.

For *unsymmetrical horizontal* square, rectangular, or circular surfaces, the characteristic length L can be calculated from the expression $L = A/P$, where A is the area and P is the wetted perimeter of the surface.

Free Convection from Spheres

For free convection from spheres, the following correlation has been developed:

$$\overline{\text{Nu}}_D = 2 + 0.43(\text{Gr}_D\,\text{Pr})^{0.25} \qquad \text{for } 1 < \text{Gr}_D < 10^5$$

Although this expression was designed primarily for gases, $\text{Pr} \approx 1$, it may be used to approximate the values for liquids as well.[15]

Free Convection in Enclosed Spaces

Heat transfer in an enclosure occurs in a number of different situations and with a variety of configurations. When a temperature difference is imposed on two opposing walls that enclose a space filled with a fluid, convective heat transfer will occur. For small values of the Rayleigh number, the heat transfer may be dominated by conduction, but as the Rayleigh number increases, the contribution made by free convection will increase. Following are a number of correlations, each designed for a specific geometry. For all of these, the fluid properties are evaluated at the average temperature of the two walls.

Cavities between Two Horizontal Walls at Temperatures T_1 and T_2 Separated by Distance δ(T_1 for Lower Wall, $T_1 > T_2$)

$$q'' = \bar{h}(T_1 - T_2)$$

$$\overline{Nu}_\delta = 0.069\text{Ra}_\delta^{1/3}\,\text{Pr}^{0.074} \qquad \text{for } 3 \times 10^5 < \text{Ra}_\delta < 7 \times 10^9$$

$$= 1.0 \qquad \text{for } \text{Ra}_\delta < 1700$$

where $\text{Ra}_\delta = g\beta(T_1 - T_2)\delta^3/\alpha\nu$ and δ is the thickness of the space.[16]

Table 43.15 Constants for Free Convection from Flat Plates and Cylinders

Geometry	$Gr_K Pr$	C	m	L
Vertical flat plates and cylinders	10^{-1}–10^{4}	Use Fig. 43.12	Use Fig. 43.12	Height of plates and cylinders; restricted to $D/L \geq 35/Gr_L^{1/4}$ for cylinders
	10^{4}–10^{9}	0.59	¼	
	10^{9}–10^{13}	0.10	⅓	
Horizontal cylinders	0–10^{-5}	0.4	0	Diameter D
	10^{-5}–10^{4}	Use Fig. 43.13	Use Fig. 43.13	
	10^{4}–10^{9}	0.53	¼	
	10^{9}–10^{13}	0.13	⅓	
Upper surface of heated plates or lower surface of cooled plates	2×10^{4}–8×10^{6}	0.54	¼	Length of a side for square plates, the average length of the two sides for rectangular plates
	8×10^{6}–10^{11}	0.15	⅓	
Lower surface of heated plates or upper surface of cooled plates	10^{5}–10^{11}	0.58	⅕	0.9D for circular disks

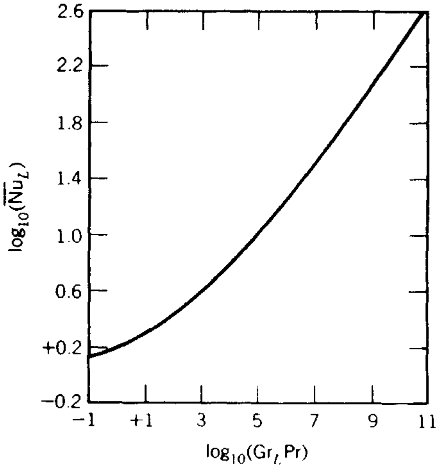


Fig. 43.13 Free convection heat-transfer correlation for heated vertical plates and cylinders. Adapted from Ref. 14. (Used with permission of McGraw-Hill Book Company.)

Cavities between Two Vertical Walls of Height H at Temperatures by Distance T_1 and T_2 Separated by Distance δ[17,18]

$$q'' = soh(T_1 - T_2)$$

$$\overline{Nu}_\delta = 0.22\left(\frac{Pr}{0.2 + Pr}\,Ra_\delta\right)^{0.28}\left(\frac{\delta}{H}\right)^{0.25}$$

for $2 < H/\delta < 10$, $Pr < 10^5$ $Ra_\delta < 10^{10}$;

$$\overline{Nu}_\delta = 0.18\left(\frac{Pr}{0.2 + Pr}\,Ra_\delta\right)^{0.29}$$

for $1 < H/\delta < 2$, $10^3 < Pr < 10^5$, and $10^3 < Ra_\delta Pr/(0.2 + Pr)$;

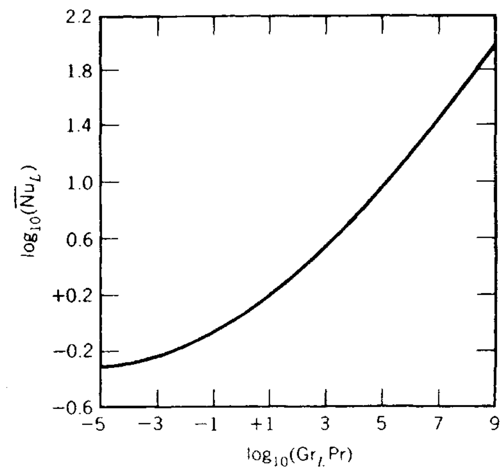


Fig. 43.14 Free convection heat-transfer correlation from heated horizontal cylinders. (Adapted from Ref. 14. Used with permission of McGraw-Hill Book Company.)

$$\overline{Nu}_\delta = 0.42 Ra_\delta^{0.25} Pr^{0.012} (\delta/H)^{0.3}$$

for $10 < H/\delta < 40$, $1 < Pr < 2 \times 10^4$, and $10^4 < Ra_\delta < 10^7$.

43.3.4 The Log Mean Temperature Difference

The simplest and most common type of heat exchanger is the *double-pipe heat exchanger,* illustrated in Fig. 43.15. For this type of heat exchanger, the heat transfer between the two fluids can be found by assuming a constant overall heat transfer coefficient found from Table 43.8 and a constant fluid specific heat. For this type, the heat transfer is given by

$$q = UA\,\Delta T_m$$

where

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

In this expression, the temperature difference, ΔT_m, is referred to as the *log-mean temperature difference* (LMTD); ΔT_1 represents the temperature difference between the two fluids at one end and ΔT_2 at the other end. For the case where the ratio $\Delta T_2/\Delta T_1$ is less than two, the *arithmetic mean temperature difference* $(\Delta T_2 + \Delta T_1)/2$ may be used to calculate the heat-transfer rate without introducing any significant error. As shown in Fig. 43.15,

$$\Delta T_1 = T_{h,i} - T_{c,i} \qquad \Delta T_2 = T_{h,o} - T_{c,o} \qquad \text{for parallel flow}$$
$$\Delta T_1 = T_{h,i} - T_{c,o} \qquad \Delta T_2 = T_{h,o} - T_{c,i} \qquad \text{for counterflow}$$

Cross-Flow Coefficient

In other types of heat exchangers, where the values of the overall heat transfer coefficient, U, may vary over the area of the surface, the LMTD may not be representative of the actual average temperature difference. In these cases, it is necessary to utilize a correction factor such that the heat transfer, q, can be determined by

$$q = UAF\,\Delta T_m$$

Here the value of ΔT_m is computed assuming counterflow conditions, $\Delta T_1 = T_{h,i} - T_{c,i}$ and $\Delta T_2 = T_{h,o} - T_{c,o}$. Figures 43.16 and 43.17 illustrate some examples of the *correction factor*, F, for various multiple-pass heat exchangers.

43.4 RADIATION HEAT TRANSFER

Heat transfer can occur in the absence of a participating medium through the transmission of energy by electromagnetic waves, characterized by a wavelength, λ, and frequency, ν, which are related by $c = \lambda\nu$. The parameter c represents the velocity of light, which in a vacuum is $c_o = 2.9979 \times 10^8$ m/sec. Energy transmitted in this fashion is referred to as *radiant energy* and the heat transfer process that occurs is called *radiation heat transfer* or simply *radiation.* In this mode of heat transfer, the energy is transferred through electromagnetic waves or through photons, with the energy of a photon being given by $h\nu$, where h represents Planck's constant.

In nature, every substance has a characteristic wave velocity that is smaller than that occurring in a vacuum. These velocities can be related to c_o by $c = c_o/n$, where n indicates the refractive index. The value of the refractive index n for air is approximately equal to 1. The wavelength of the energy given or for the radiation that comes from a surface depends on the nature of the source and various wavelengths sensed in different ways. For example, as shown in Fig. 43.18 the electromagnetic spectrum consists of a number of different types of radiation. Radiation in the visible spectrum occurs in the range $\lambda = 0.4\text{–}0.74\ \mu m$, while radiation in the wavelength range $0.1\text{–}100\ \mu m$ is classified as *thermal radiation* and is sensed as heat. For radiant energy in this range, the amount of energy given off is governed by the temperature of the emitting body.

43.4.1 Black-Body Radiation

All objects in space are continuously being bombarded by radiant energy of one form or another and all of this energy is either absorbed, reflected, or transmitted. An ideal body that absorbs all the radiant energy falling upon it, regardless of the wavelength and direction, is referred to as a *black body.* Such a body emits the maximum energy for a prescribed temperature and wavelength. Radiation from a black body is independent of direction and is referred to as a *diffuse emitter.*